



## Evaluation the Collisional Stopping power and Continuous Slow Down Approximation ranges for protons in Carbon

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### Abstract

The stopping power and CSDA ranges of energetic protons in carbon has been calculated using a model based on different theoretical approaches within energy range (1.5 -295)MeV . The present work accounts for the incident proton slows down due to the inelastic collision between target electrons of the medium and the incident proton moving through it . These interactions of the incident proton with the target's electrons is a function of the proton energy . Our results are in good agreement available data of the universal code pstar . This calculation model can be useful for different applications in nuclear physics, medical physics , materials science and space radiation health.

### 1.Introduction

The heavy charged particles like proton interact with matter primarily through coulomb forces between positive and negative charge of the orbital electrons within the absorber atoms .Protons interacts with matter in three distinct ways<sup>[1-3]</sup>:

- Proton slow down by myriad collisions with atomic electrons.
- Protons are deflected by myriad collisions with atomic nuclei.
- Proton are sometimes have a head-on collision with a nucleus, setting secondary particles in motion .

These three processes are named as stopping, scattering, and nuclear interactions respectively .Stopping and scattering proceed via the electromagnetic (EM) interaction between the charge of the proton and the charge of atomic electrons or nucleus, as the case may be. Thus, the protons gradually lose energy in many small steps<sup>[3]</sup>.The average energy loss of the proton per unit path length is mainly characterized by the stopping power. The stopping power process has been a test bed for the development of many new theoretical approaches, with many papers having been written estimating the proton stopping power both theoretically and experimentally. Molina et al<sup>[4]</sup> have applied a Monte-Carlo code combined with a finite differences algorithm, evaluated in the dielectric framework to calculate the stopping power of proton beams

for incident energies range (0.5–10 MeV/u) in liquid water as a function of the target depth. Paul’s work [5] gave an overview of results of the stopping power for positive ions, obtained during the last few years, and discussed the new experimental stopping power results obtained for low energy protons in metals and in liquid water. Ziegler, Biersack, and Littmark developed a semi-empirical model, called ZBL stopping, to calculate the stopping power of different ions in matter. On the other hand, many computer simulation programs have been developed since the 1960 [6-12] most of which are based on the binary collision approximation, in which the ion is approximated to travel through a material by experiencing a sequence of independent collisions with electron targets, but loses no energy in collisions with nuclei. Eppacher et al. [13] did measurements of the stopping power of rubidium and strontium for protons in order to check the quality of the several available interpolations [14–16], and they found differences up to 30% between interpolated data and their measurements.

The aim of our study is to calculate the collisional stopping power  $S_{col}$  and ranges  $R_{CSDA}$  of carbon in the energy range (1.5-295)MeV and compared with results of pstar [16].

## 2.Theoretical part

### 2.1Collisional Stopping power of low energy proton

When a charged particle (CP)travels inside a stopping medium ,it interacts with the atoms of the absorber and gradually loses its kinetic energy in a large number of small steps. The mean rate of energy loss per unit path length of the CP of kinetic energy  $T_p$  traversing an absorbing medium of atomic number  $Z$  is defined as the stopping power of the absorbing medium. Stopping power depends on physical properties of the absorbing medium as well on properties of the CP traversing the absorbing medium. Stopping power can be considered as a property of the absorbing medium in which a CP propagates. Stopping power has an important role in many facets of basic science and technology, and is used heavily in clinical radiation dosimetry based on ionization chambers [17].The theoretical rate of energy loss of a fast charged particle in matter was derived by Bethe and Bloch 1932[18].Good accounts with references can be found in the range–energy tables of Janni [19] and ICRU Report 49 [20] .The mass collisional stopping power in an elementary material of atomic number  $Z$  and relative atomic mass  $A$  is given by [20-21]

$$\frac{S_{col}}{\rho} = \frac{2\pi r_e^2 N_A m_e c^2}{A^2} \frac{Z}{\beta^2} \left( \ln \frac{2m_e c^2 \beta^2}{I^2 (1-\beta^2)} - 2\beta^2 \right) \dots\dots\dots(1)$$

Where the terms are defined in the following table below.

Table-1: Values and definitions of the terms employed in the equation (1).

Terms and Values	Definition
$m_e c^2 = 0.511 MeV$	Rest mass energy of electron
$r_e^2 = 2.81798 \times 10^{-13} cm$	Classical radius of electron
$N_A = 6.023 \times 10^{23} mol^{-1}$	Avogadro's number
$Z$ and $A$	The atomic number and atomic weight of target material
$z$	Atomic number of incident projectile
$I$	Mean ionization energy of target material

After substitution the above values , the expression(1), simplified to

$$\frac{S_{col}}{\rho} = 0.3072 \frac{Z}{A^2} \frac{z^2}{\beta^2} \left( \ln \frac{W_m}{I} - \beta^2 \right) \dots\dots\dots(2)$$

Where

$$W_m = \frac{1.022\beta^2}{1 - \beta^2} \dots\dots\dots(3)$$

and

$$\beta^2 = 1 - \frac{1}{\left( 1 + \frac{T_p}{938.3} \right)^2} \dots\dots\dots(4)$$

Here  $\beta^2$  is the largest possible proton energy loss in a single collision with a free electron. This equation has been corrected for two factors at very high and moderately low energies. One is the shielding of distant electrons because of the polarization of electrons by the electric field of the moving ion. This effect depends of the electron density and becomes more and more important as the energy of incident particle increases. The second correction term applies at lower energies and depends on the orbital velocities of the electrons. Both of these correction terms are subtractive and generally represented by the symbols  $\delta$  and  $C$  respectively[21]but ,due to smallness of these corrections ,we ignored it.

**2.2 Mean Ionization parameter  $I$**

Mean Ionization parameter  $I$  of an atom or molecule describes the average minimum amount of energy required to remove an electron from certain electron shell to infinity which measured in eV [21].A number of semi-empirical representations of  $I$  as a function of the medium's atomic number have been provided in the literature. A particularly useful one is given by[22]:

$$I(eV) = 12Z + 7, Z < 13 \dots \dots \dots (5)$$

$$I(eV) = 9.76Z + 5.58Z^{-0.19}, Z \geq 13 \dots \dots \dots (6)$$

The ionization potential  $I$  of the medium represents one of the difficult parameters to evaluate. The average parameterization of  $I$ , normalized to the atomic number of the medium.

**2.3 Continuous Slow Down Approximation Range of protons ( $R_{CSDA}$ )**

The abbreviation CSDA stands for the Continuous Slow Down Approximation .Under this approximation, the proton which penetrates into the absorbers ,loos energy continuously by inelastic collision ,and the total path length which the particle would travel in the absorber during slowing down process in an unbounded homogenous medium from initial energy to the final kinetic energy zero is called  $R_{CSDA}$  range of the particle[23].For carbon, it can be approximately given by[24]:

$$R_{CSDA}(g/cm^2) = \frac{T_p^{1.77}}{415} + \frac{1}{670}, 1MeV < T_p < 300MeV \dots \dots \dots (7)$$

So it shows that the  $R_{CSDA}$  ranges of materials can be expressed in terms of energy and independent of atomic number of the material.

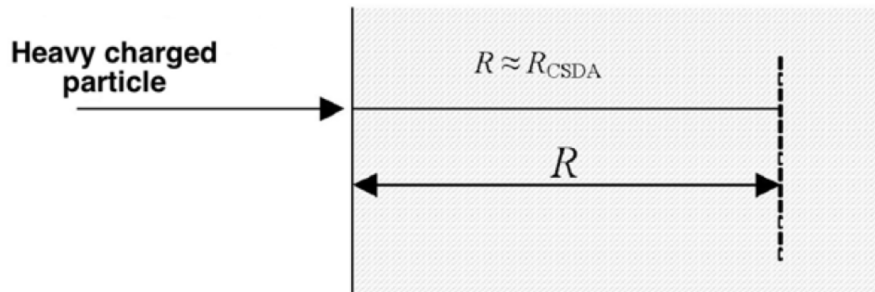


Fig.(1):Schematic diagram of a Heavy charged particle like a proton penetration into a medium[25].

**3.Results and discussion**

The collisional stopping power of proton have been calculated for carbon from equation (2) .Table 2 and fig.(2)shows the plotted stopping power via energies for our study in comparison to that of Ref.[16] interpretation of these results indicates that the increasing collisional stopping power with decreasing proton energy. This behavior can be explained that, a slow particle (lower energy proton ) spends more time in the proximity of the target, hence has a higher probability of interaction atomic electrons , while a swift protons (Higher energy) can sweep through the target or its potential field without being affected much more . This means that the protons of low kinetic energy ,has highly probability of interactions with atomic electrons and low probability to interactions with atomic electrons for higher energetic protons. The dependence of  $S_{col}$  on the absorbing medium , which is  $S_{col}$  depends on atomic number  $Z$  of the absorber in two ways: directly through the electron density  $N_e = N_A Z / A$  of the absorber where  $S_{col}$  is proportional to  $Z / A$  and indirectly through the mean ionization/ excitation potential  $I$  of the absorber as in equation 5 .On the other hand , we note the dependence of  $S_{col}$  on physical characteristic of the charged particle:

- As shown in equation 2,  $S_{col}$  depends on CP velocity  $v$  and charge  $ze$  but does not depend either directly or indirectly on the rest mass  $m_e c^2$  of the CP. So it means a given absorbing material will have the same  $S_{col}$  for all heavy CPs of a given kinetic energy  $T_p$  and charge  $ze$ .

- In eq.(1), that  $S_{col}$  is linearly proportional to  $z^2$  where  $ze$  stands for the charge of the CP. For example,  $z = 1$  for proton and deuteron;  $z = 2$  for  $\alpha$  particle .

- As evident from equations 1 and 2,  $S_{col}$  depends on CP velocity  $\beta = v/c$  through the  $1/\beta^2$  .

-For proton which is heavy CPs the radiative stopping power  $S_{rad}$  is much smaller than the mass collision stopping power  $S_{col}$ , so that the total mass stopping power  $S_{tot}$  is roughly equal to  $S_{col}$  .

Table (3) shows that the  $R_{CSDA}$  range (or mean path-length) in the carbon (see ranges table 2 and figure 2) is a function of kinetic energy of the proton . The incoming proton's spend less time near the orbital electrons, thus reducing the effect of Coulomb interactions (consequently stopping power) and increasing the range. All results of present works are in good agreement and, the error noted for  $S_{col}$  about 5% and for  $R_{CSDA}$  8% at  $T_p = 2.5MeV$  , this small differences may not lead to wrong interpretation of the results . The error percentage in our calculation can be estimated from [ 26 ] :

$$Error\% = \left| \frac{presentwork - pstar}{presentwork} \right| 100\%, \dots \dots \dots (8)$$

#### 4. Concluding Remarks

Since  $S_{col}$  originates with inelastic collisions between the CP and orbital electrons of the absorber atoms, a conclusion can be made that  $S_{col}$  is proportional to the electron density  $N_e$  given as the number of electrons per mass of the absorber or  $N_e / m = \frac{ZN_A}{A}$ . The dependence of  $S_{col}$  on the absorbing medium,  $S_{col}$  depends on atomic number  $Z$  intensively . For proton which is heavy CPs the radiation stopping power  $S_{rad}$  is much smaller than the mass collision stopping power  $S_{col}$ , so that the total mass stopping power  $S_{tot}$  is roughly equal to  $S_{col}$ . We come to the conclusion that the energy of the incident proton is the key parameter for the calculation of CSDA range

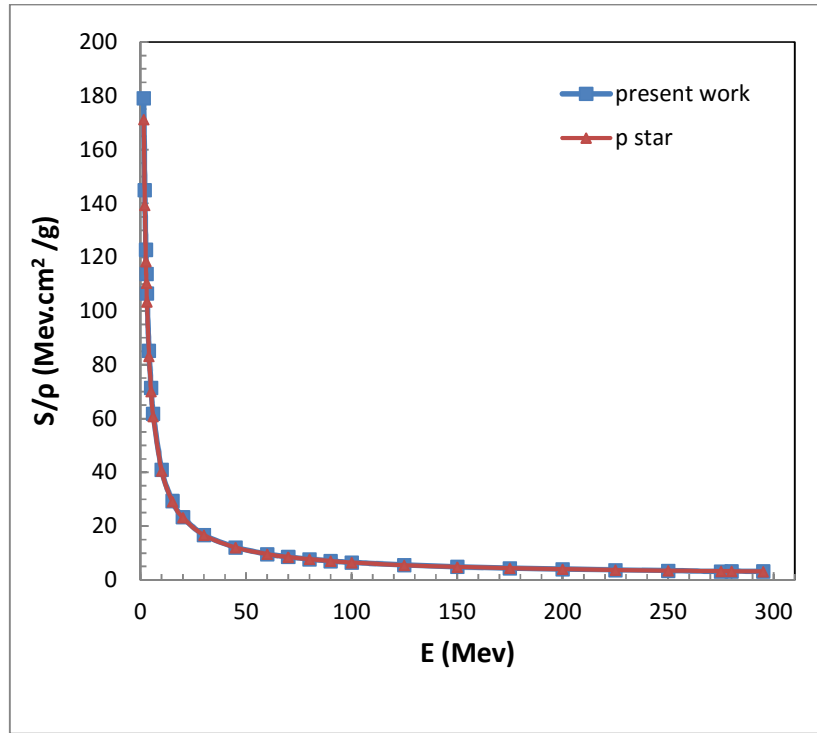


Fig.(2):A comparison of collision stopping power of present work with the results of reference [16].

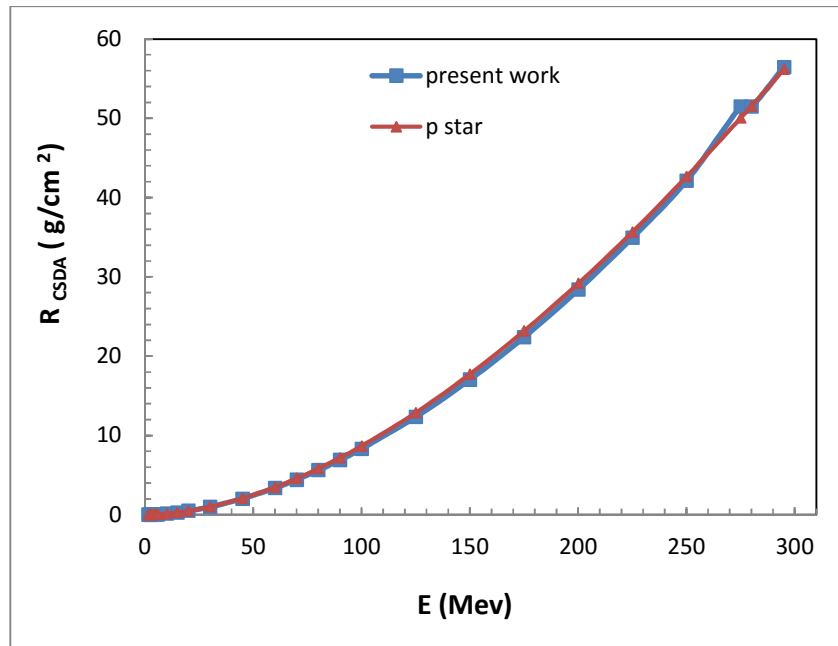


Fig.(3):A comparison of  $R_{CSDA}$  ranges of present work with the results of reference [16].

Table -2: Comparison of results for collisional stopping power of proton by using equation (2) with Ref.[16] in Carbon .

$T_p$ (MeV)	Present work	Ref.[16]	Error%	$T_p$ (MeV)	Present work	Ref.[16]	Error%
1.5	179.158	171.2	4.441	60	9.657	9.591	0.6834
2.5	122.818	118.5	3.515	90	7.0666	7.02	0.660
2.75	113.86	110.4	3.05	100	6.53	6.486	0.674
3	106.635	103.5	2.94	125	5.55	5.50	0.90
4	85.221	83.26	1.838	150	4.87	4.844	0.54
5	71.49	70.1	1.958	175	4.4	4.36	0.90
6	61.804	60.81	1.60	200	4.024	3.994	0.075
10	41.047	40.55	1.21	225	3.74	3.7	1.0695
15	29.433	29.25	0.6217	250	3.50	3.475	0.7142
20	23.374	23.17	0.872	275	3.13	3.284	4.92
30	16.78	16.68	0.60	280	3.28	3.25	0.9145
45	12.13	12.04	0.741	295	3.189	3.155	1.066

Table-3: Comparison of results  $R_{CSDA}$  Ranges of Proton in carbon by using equation (7) with Ref.[16] in Carbon .

$T_p$ (MeV)	Present work	Ref.[16]	Error%	$T_p$ (MeV)	Present work	Ref.[16]	Error%
1.5	0.00642	0.0054	1.588	60	3.3707	3.477	3.154
2.5	0.01365	0.01256	7.985	90	6.9074	7.189	4
2.75	0.01588	0.01475	7.115	100	8.323	8.672	4.20
3	0.0183	0.01709	6.6120	125	12.353	12.87	4.18
4	0.0294	0.02973	1.2224	150	17.058	17.73	3.94
5	0.04294	0.04107	4.22	175	22.408	23.18	3.44
6	0.05872	0.05642	3.9168	200	28.383	29.17	2.77
10	0.1428	0.1388	2.8	225	34.962	35.64	1.939
15	0.2912	0.2862	1.717	250	42.128	42.65	1.24
20	0.4835	0.4796	0.8066	275	51.486	50.05	2.78
30	0.9894	0.996	0.667	280	51.486	51.58	0.1825
45	2.0264	2.071	2.2	295	56.4687	56.21	0.4581

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